University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

# ASSIGNMENT 5

**Reading:**
105 Notes 3.4-3.7
Hand & Finch 3.4-3.9

**1.**
A particle of mass $m$ and electric charge $q$ is situated in an alternating electric field directed along the $x$ axis: $E_x = E_0 \cos\omega t$. The particle also experiences a force in the $x$ direction proportional to the *third derivative* of its $x$ coordinate:

$$F_\alpha = +\alpha \frac{d^3x}{dt^3} \ ,$$

where $\alpha$ is a positive constant. [This model gives an approximate description of a charged particle that scatters radiation.]

Find the amplitude and phase of the particle's oscillation in the steady state.

**2.**
Consider an extremely underdamped oscillator ($\omega_0/\gamma \equiv Q \gg 1$).
(**a**)
Suppose that the oscillator is undriven, but initially it is excited. How many oscillation periods are required for the energy stored in the oscillator to diminish by a factor of $e$?
(**b**)
Instead suppose that the oscillator is driven at resonance. What is the ratio of the energy stored in the oscillator to the work done by the driving force in one oscillation period?

**3.**
Consider an undriven oscillator satisfying the initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$. Find $x(t)$ when the oscillator is...
(**a**)
...slightly underdamped ($\omega_0/\gamma \equiv Q = \frac{1}{\sqrt{2}}$).
(**b**)
...slightly overdamped ($\omega_0/\gamma \equiv Q = \frac{6}{13}$).

**4.**
Consider a critically damped oscillator ($\omega_0/\gamma \equiv Q = \frac{1}{2}$) that remains at rest at $x = 0$ for $t < 0$, but is driven at resonance by a force $F_x$ such that

$$\frac{F_x}{m} = G \sin\omega_0 t$$

for $t > 0$, where $G$ is a constant. Find $x(t)$.
[*Hint:* It is somewhat easier to solve this problem directly (matching boundary conditions at $t = 0$) than to use a Green function.]

**5.**
*Woofer design.* With compact discs well established as a recording medium, loudspeaker distortion is the last major barrier to true sound reproduction. A woofer in a sealed box ("acoustic suspension") is the simplest type to analyze. The motion of the cone of mass $m$ is governed by the equation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos\omega t \ ,$$

where $F_0$ is constant if the amplifier output resistance or the voice coil resistance is excessive (not a typical assumption, but one we will make here for simplicity). The average sound intensity is proportional to the average (acceleration)$^2$ of the cone. The damping factor $b$ is proportional to the strength of the magnetic "motor" – the magnet and voice coil assembly. The spring constant $k$ is inversely proportional to the volume of air sealed in the box.
(**a**)
Try to think up a simple mechanical test that you can perform in the showroom (when the salesperson is looking the other way) to see whether

the cone is underdamped or overdamped.

(**b**)

Suppose the assembly goes through resonance at $\nu_0 = 50$ Hz with $Q = 1$. (These are typical specifications for a medium quality classical music speaker.) By what factor will the sound intensity vary at 25 Hz? 100 Hz?

(**c**)

Sketch the effect upon smoothness of bass response of greatly increasing the cone area (to an inexperienced buyer, this often increases the speaker's apparent value). [*Hint*: Make reasonable assumptions concerning the dependence of $m$ and $k$ on the cone area.]

**6.**

Obtain the Fourier series that represents the function

$$F(t) = 0 \ \ (-\frac{2\pi}{\omega} < t < 0)$$
$$= F_0 \sin \omega t \ \ (0 < t < \frac{2\pi}{\omega}) \ .$$

**7.**

Consider a damped oscillator (as usual, characterized by $\gamma$, $\omega_0$, and $m$) driven by a periodic force $F$. During one period,

$$-\frac{2\pi}{\omega} < t < \frac{2\pi}{\omega} \ ,$$

$F$ is taken to be equal to $F(t)$ in problem (**6.**); before and afterward, it simply repeats itself.

Find $x(t)$ for this oscillator. You may assume that any transient effects, due to the driving force having been turned on at $t = -\infty$, have damped out.

**8.**

Derive the Green function for an *overdamped* oscillator initially at rest at the origin. [*Hint*: Use the method of 105 Notes sections 3.5 and 3.6.]